The point of this proof is to demonstrate that the potential correlation coefficient (denoted by P if we are speaking of the population parameter) is equal to 1 under the assumption that signal percepts are independent and identically distributed, and that noise percepts are independent and identically distributed. This is an assumption held by Signal Detection Theory as well as other theories of decision making under uncertainty. In Potential Performance Theory (PPT; see Trafimow & Rice, 2009 for details), the potential correlation coefficient is defined as

$$\mathrm{P} = \frac{\rho_\phi}{\sqrt{\rho_{XX'}}}$$

ρ_ϕ (the uncorrected correlation coefficient) is defined as

$$\rho_\phi = \frac{a_\phi d_\phi - b_\phi c_\phi}{\sqrt{(a_\phi + b_\phi)(c_\phi + d_\phi)(a_\phi + c_\phi)(b_\phi + d_\phi)}}$$

The constants in the above equation are taken from the following contingency table:

		Participant Response (X)	
		X = 1	X = 0
Stimulus (Y)	Y = 1	a_ϕ	b_ϕ
	Y = 0	c_ϕ	d_ϕ

$\rho_{XX'}$ (the consistency correlation coefficient) is defined as the correlation between responses associated with two identical blocks of trials (the same stimuli in the same order). The formula for this parameter is:

$$\rho_{XX'} = \frac{a_{XX'} d_{XX'} - b_{XX'} c_{XX'}}{\sqrt{(a_{XX'} + b_{XX'})(c_{XX'} + d_{XX'})(a_{XX'} + c_{XX'})(b_{XX'} + d_{XX'})}}$$

The constants in the above equation are taken from the following contingency table (see also Table 1 in the main manuscript):

		Response in Block 1 (X)	
		X = 1	X = 0
Response in Block 2 (X’)	X’ = 1	$a_{XX'}$	$b_{XX'}$
	X’ = 0	$c_{XX'}$	$d_{XX'}$

Let *n* be the number of trials in a block. We will start the proof by transforming ρ_ϕ:

$$\rho_\phi = \frac{[n \cdot p(X=1 \,\&\, Y=1)][n \cdot p(X=0 \,\&\, Y=0)] - [n \cdot p(X=1 \,\&\, Y=0)][n \cdot p(X=0 \,\&\, Y=1)]}{\sqrt{[n \cdot p(Y=1)][n \cdot p(Y=0)][n \cdot p(X=1)][n \cdot p(X=0)]}}$$
$$= \frac{p(X=1 \,\&\, Y=1)p(X=0 \,\&\, Y=0) - p(X=1 \,\&\, Y=0)p(X=0 \,\&\, Y=1)}{\sqrt{p(Y=1)p(Y=0)p(X=1)p(X=0)}}$$
$$= \frac{p(X=1|Y=1)p(Y=1)p(X=0|Y=0)p(Y=0) - p(X=1|Y=0)p(Y=0)p(X=0|Y=1)p(Y=1)}{\sqrt{p(Y=1)p(Y=0)p(X=1)p(X=0)}}$$
$$= \frac{p(Y=1)p(Y=0)[p(X=1|Y=1)p(X=0|Y=0) - p(X=1|Y=0)p(X=0|Y=1)]}{\sqrt{p(Y=1)p(Y=0)p(X=1)p(X=0)}}$$

Let $Q = p(X=1|Y=1)p(X=0|Y=0) - p(X=1|Y=0)p(X=0|Y=1)$.

Then

$$\rho_\phi = \frac{p(Y=1)p(Y=0)Q}{\sqrt{p(Y=1)p(Y=0)p(X=1)p(X=0)}}$$

$$= Q\sqrt{\frac{p(Y=1)p(Y=0)}{p(X=1)p(X=0)}} \tag{1}$$

Let $a_{XX'}$, $b_{XX'}$, $c_{XX'}$, and $d_{XX'}$ be the expected cell frequencies for the first through fourth cells of the XX' contingency table, respectively.

$$a_{XX'} = n \cdot p(X=1 \,\&\, X'=1)$$

$$= n \cdot [p(X=1 \,\&\, X'=1|Y=1)p(Y=1) + p(X=1 \,\&\, X'=1|Y=0)p(Y=0)] \tag{2}$$

If we assume that $(X|Y=1)$ and $(X'|Y=1)$ are independent and identically distributed (iid) and that $(X|Y=0)$ and $(X'|Y=0)$ are also iid, then Equation 2 reduces to

$$a_{XX'} = n \cdot [p(X=1|Y=1)p(X'=1|Y=1)p(Y=1) + p(X=1|Y=0)p(X'=1|Y=0)p(Y=0)]$$

$$= n \cdot [p(X=1|Y=1)^2 p(Y=1) + p(X=1|Y=0)^2 p(Y=0)] \tag{3}$$

Using the same procedure, the formulas for the remaining cell frequencies are

$$b_{XX'} = c_{XX'} =$$

$$n \cdot [p(X=0|Y=1)p(X=1|Y=1)p(Y=1) + p(X=0|Y=0)p(X=1|Y=0)p(Y=0)] \tag{4}$$

$$d_{XX'} = n \cdot [p(X=0|Y=1)^2 p(Y=1) + p(X=0|Y=0)^2 p(Y=0)] \quad (5)$$

Combining Equations 3 and 5, then,

$$a_{XX'} \cdot d_{XX'} =$$
$$\left[n \cdot [p(X=1|Y=1)^2 p(Y=1) + p(X=1|Y=0)^2 p(Y=0)]\right]\left[n \cdot [p(X=0|Y=1)^2 p(Y=1) + p(X=0|Y=0)^2 p(Y=0)]\right] =$$
$$n^2[p(X=1|Y=1)^2 p(X=0|Y=1)^2 p(Y=1)^2 + p(X=1|Y=0)^2 p(X=0|Y=1)^2 p(Y=1)p(Y=0) + p(X=1|Y=0)^2 p(X=0|Y=0)^2 p(Y=0)^2] \quad (6)$$

Similarly, squaring Equation 3 yields

$$b_{XX'} \cdot c_{XX'} =$$
$$n^2[p(X=0|Y=1)^2 p(X=1|Y=1)^2 p(Y=1)^2 +$$
$$2p(X=1|Y=1)p(X=1|Y=0)p(X=0|Y=1)p(X=0|Y=0)p(Y=1)p(Y=0) +$$
$$p(X=0|Y=0)^2 p(X=1|Y=0)^2 p(Y=0)^2] \quad (7)$$

Combining Equations 6 and 7 and simplifying terms yields

$$a_{XX'} \cdot d_{XX'} - b_{XX'} \cdot c_{XX'} =$$

$$n^2 \begin{bmatrix} p(X=1|Y=1)^2 p(X=0|Y=1)^2 p(Y=1)^2 + p(X=1|Y=0)^2 p(X=0|Y=1)^2 p(Y=1)p(Y=0) \\ +p(X=1|Y=0)^2 p(X=0|Y=0)^2 p(Y=0)^2 \end{bmatrix}^2$$
$$-n^2 \begin{bmatrix} p(X=0|Y=1)^2 p(X=1|Y=1)^2 p(Y=1)^2 \\ +2p(X=1|Y=1)p(X=1|Y=0)p(X=0|Y=1)p(X=0|Y=0)p(Y=1)p(Y=0) \\ +p(X=0|Y=0)^2 p(X=1|Y=0)^2 p(Y=0)^2 \end{bmatrix}$$
$$= n^2 p(Y=1)p(Y=0) \begin{bmatrix} p(X=1|Y=1)^2 p(X=0|Y=0)^2 \\ -2p(X=0|Y=1)p(X=1|Y=1)p(X=0|Y=0)p(X=1|Y=0) \\ +p(X=1|Y=0)^2 p(X=0|Y=1)^2 \end{bmatrix}$$
$$= n^2 p(Y=1)p(Y=0) \begin{bmatrix} p(X=1|Y=1)^2 p(X=0|Y=0)^2 \\ -p(X=0|Y=1)p(X=1|Y=1)p(X=0|Y=0)p(X=1|Y=0) \\ +p(X=1|Y=0)^2 p(X=0|Y=1)^2 \\ -p(X=0|Y=1)p(X=1|Y=1)p(X=0|Y=0)p(X=1|Y=0) \end{bmatrix}$$
$$= n^2 p(Y=1)p(Y=0)[p(X=1|Y=1)p(X=0|Y=0) - p(X=1|Y=0)p(X=0|Y=1)]^2$$
$$= n^2 p(Y=1)p(Y=0)Q^2$$

Therefore,

$$\rho_{XX'} = \frac{a_{XX'} \cdot d_{XX'} - b_{XX'} \cdot c_{XX'}}{\sqrt{[n \cdot p(X=1)][n \cdot p(X=0)][n \cdot p(X'=1)][n \cdot p(X'=0)]}}$$
$$= \frac{n^2 p(Y=1)p(Y=0)Q^2}{n^2\sqrt{p(X=1)p(X=0)p(X'=1)p(X'=0)}}$$

Since we assume that $(X|Y = 1)$ and $(X'|Y = 1)$ are iid and that $(X|Y = 0)$ and $(X'|Y = 0)$ are also iid, then $p(X = 1) = p(X' = 1)$ and $p(X = 0) = p(X' = 0)$, so the above equation further reduces to

$$\rho_{XX'} = \frac{p(Y=1)p(Y=0)Q^2}{\sqrt{p(X=1)^2 p(X=0)^2}} = Q^2 \frac{p(Y=1)p(Y=0)}{p(X=1)p(X=0)} \tag{8}$$

Substituting Equation 1 and 8 into the equation for P (uppercase rho), then, we get

$$\mathrm{P} = \frac{\rho_\phi}{\sqrt{\rho_{XX'}}} = \frac{Q\sqrt{\frac{p(Y = 1)p(Y = 0)}{p(X = 1)p(X = 0)}}}{\sqrt{Q^2 \frac{p(Y = 1)p(Y = 0)}{p(X = 1)p(X = 0)}}} = 1$$

Therefore, we have proven that $\mathrm{P} = 1$under the assumption that that $(X|Y = 1)$ and $(X'|Y = 1)$ are iid and that $(X|Y = 0)$ and $(X'|Y = 0)$ are also iid. Of course, the same result holds for a 2-interval forced choice (2IFC) task: a yes/no detection trial is effectively the first half of a 2IFC trial. If corrected performance is perfect for a yes/no detection task, performance will also be perfect for the associated 2IFC task.